AKADEMEIA
HIGH SCHOOL

Student's name	

academic potential test for future A-level students 2020

Time allowed: 90 minutes

Instructions:

- Use black ink or black ball-point pen.
- Answer all questions.
- You are allowed to use a calculator.
- Unless otherwise stated in a question, all numerical answers must be given either exactly or correct to 3 significant figures.

Information:

- The test consists of a few sections.
- Note that the last section is for students who consider doing Further Mathematics in A-levels.

For Examiner's Use	
Examiner's Initials	
Questions	Mark
1-4	/17
5-7	/10
8-10	/7
11-14	/13
15-17	/8
(*)18-24	/18
total Standard	/55
total Advanced	/73
SL score	
AL score	

ARITHMETIC & ALGEBRA

Q1. ***[8 marks]***
Express in terms of a. Give your answer in exact form.

Q1.1 $\sqrt[3]{a^{-2}a^{8}}$

Q1.2 $\sqrt{a^{6} \times \left(1\frac{1}{3}\right)^{-2}}$

Q1.3 $\left(\frac{1}{4}a^{-2}\right)^{3} \div \left(\frac{1}{2}a^{-3}\right)^{4}$

Q1.4 $\frac{(2a)^{8}(4a^{2})^{7}}{(16a^{3})^{8} \div (-8a)^{3}}$

Q2. ***[2 marks]***
Find the number whose 22% is equal 75. Give your answer to 2 decimal places.

Q3. ***[3 marks]***
Rationalize the denominator of the following fractions. Show your workings fully.

Q3.1 $\frac{8-4}{\sqrt{2}}$

Q3.2 $\frac{2a\sqrt{3+a}}{\sqrt{3-2}}$

Q4. ***[4 marks]***
Expand and leave the answer in simplest form.

Q4.1 $\left(4x+\frac{1}{2}\right)^2$

Q4.2 $(a-b+ab)^2$

Q4.3 $(a\sqrt{2}-3)(a\sqrt{8}+1)-(2a)(2a-2\sqrt{2})$

STATISTICS & PROBABILITY

Q5. ***[3 marks]***

A fair cubic dice and a fair coin are tossed once. On one side of the coin there is a number 1 and on the other side there is a number 2. What is the probability that the sum of outcomes is larger than 4?

Q6. ***[4 marks]***

Consider four **different** whole numbers that have the following properties:

- their range is 6,
- their median is 7,
- their mean is 7.5.

Find the numbers.

Q7. ***[3 marks]***

In a group of 20 students 14 learn French, 9 learn German and 2 do not learn any of the two languages. A student is chosen at random from the group. What is the probability that he learns both French and German?

EQUATION OF A LINE

Q8. ***[2 marks]***
Consider the points $(-3, 4)$ and $(1, -3)$.

Q8.1 Find the gradient of the line passing through the points.
Give your answer as an exact fraction.

Q8.2 Find the distance between the points.

Q9. ***[3 marks]***
Find the equation of a line perpendicular to $y = 1.5x + 2$ and passing through point $(6, -1)$.
Give your answer in the form $Ax + By + C = 0$, where A, B and C are integers.

Q10. ***[2 marks]***
Find the area of the triangle bounded by the line $y = \frac{2}{3}x - 3$ and the coordinate axes.

EQUATIONS & INEQUALITIES

Q11. ***[3 marks]***
Find the set of common solutions of the following inequalities.

$$7 - 2x > 0 \quad \text{and} \quad 5x + 10 \geq 0 \quad \text{and} \quad |x| < 1$$

Q12. ***[7 marks]***
Solve the equations and inequalities. Give all answers in simplest form.

Q12.1 $x^2 + 7x - 8 = 0$

Q12.2 $\frac{x-1}{3} = \frac{2}{x-1}$

Q12.3 $x + \frac{6}{x} = 7$

Q12.4 $x^2 = 5x$

Q12.5 $|x + 1| = 3$

Q12.6 $x^2 - 2x < 3$

Q13. ***[1 marks]***

Make r the subject of the formula $F = G\frac{m_1 m_2}{r^2}$.

Q14. ***[2 marks]***

Solve the following equations simultaneously.

$$5x + 4y = 6 \quad \text{and} \quad 3x - 2y = 8$$

TRIGONOMETRY

You may like to use the cosine rule in this section: $c^2 = a^2 + b^2 - 2ab\cos C$.

Q15. ***[3 marks]***

In triangle ABC the sides AB and BC are 6 and 4 respectively. The angle at A is 35°. Find the measure of the angle C.

Q16. ***[2 marks]***

Find the measure of the smallest angle in the triangle with sides 3, 5 and 7.

Q17. ***[3 marks]***

Q17.1 Find the obtuse angle B such that $\sin B = \sin 40^\circ$.

Q17.2 Find an angle C such that $\cos C = -\sin C$.

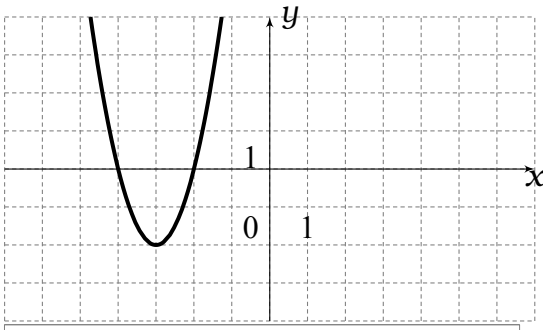
Q17.3 D is an acute angle ($0^\circ < D < 90^\circ$). Find the exact value of $\cos D$ if $\sin D = \frac{2}{3}$.

For candidates who consider doing A Level Mathematics
but not Further Mathematics the exam finishes here.
An additional section for students considering
Further Mathematics starts on the next page.

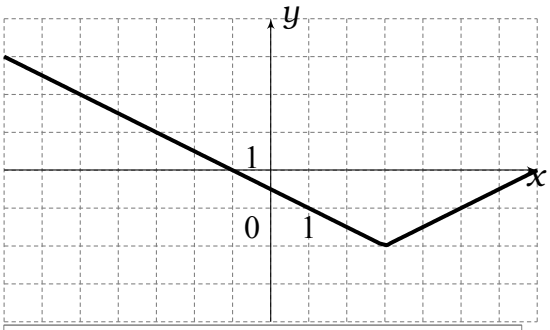
ADVANCED PART

Q18. ***[4 marks]***
Write down an equation of each of the curves shown below.

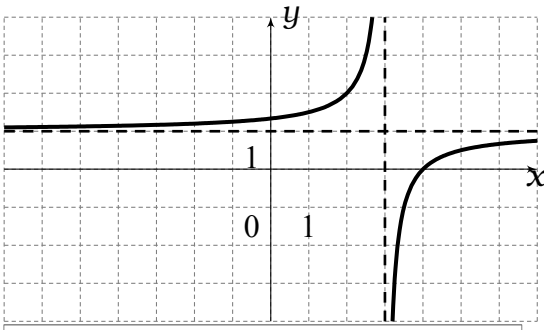
Q18.1.



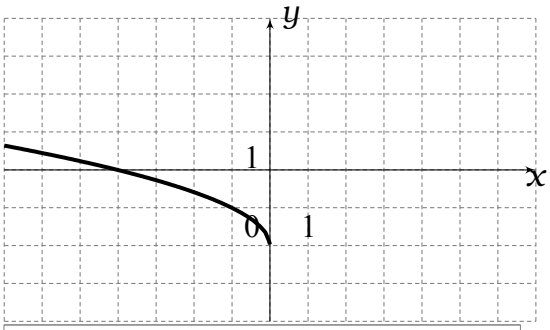
Q18.3.



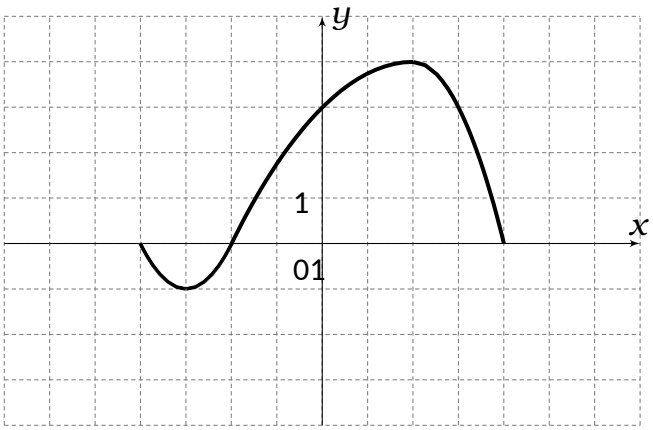
Q18.2.



Q18.4.

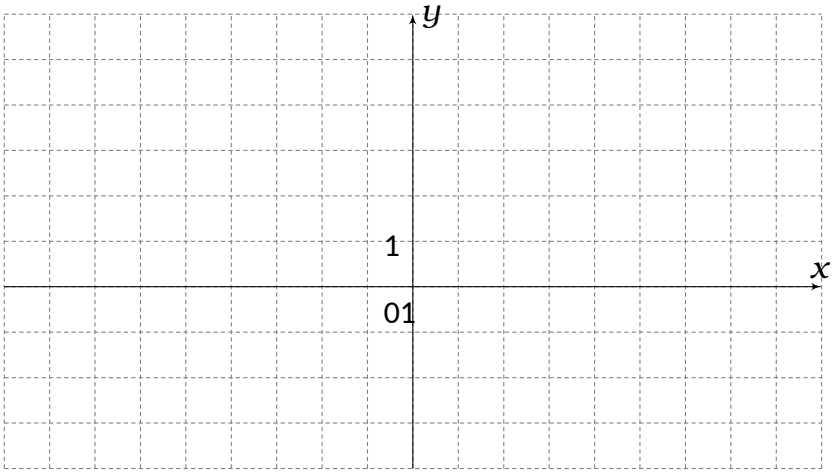


Q19. ***[3 marks]***
Consider the graph of the function $y = f(x)$ shown below.



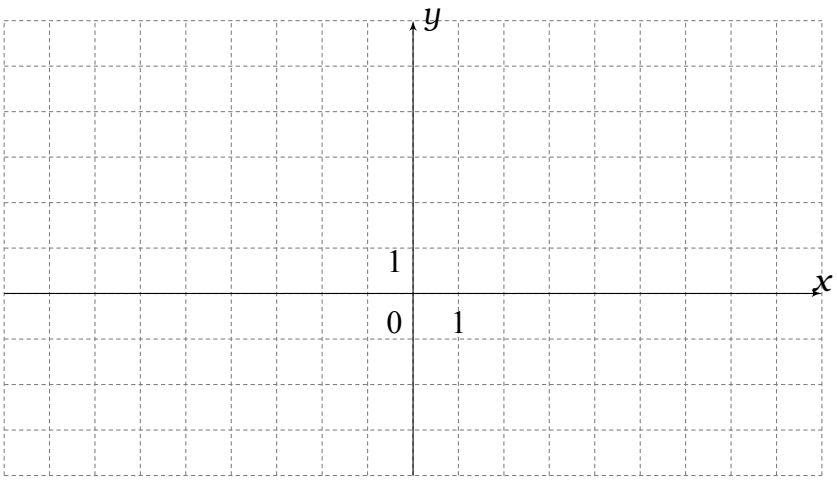
In the diagrams on the next page sketch the graphs of the curves with given equations.

Q19.1.



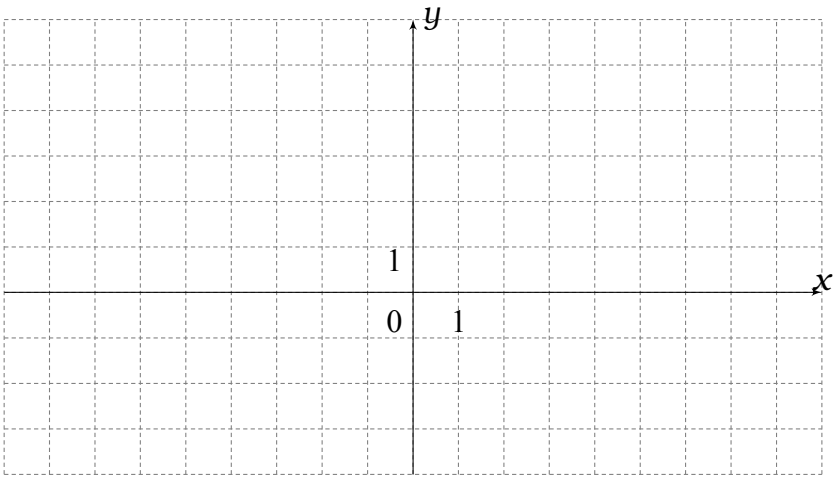
$y = \frac{1}{2}f(x)$

Q19.2.



$y = f(2x)$

Q19.3.



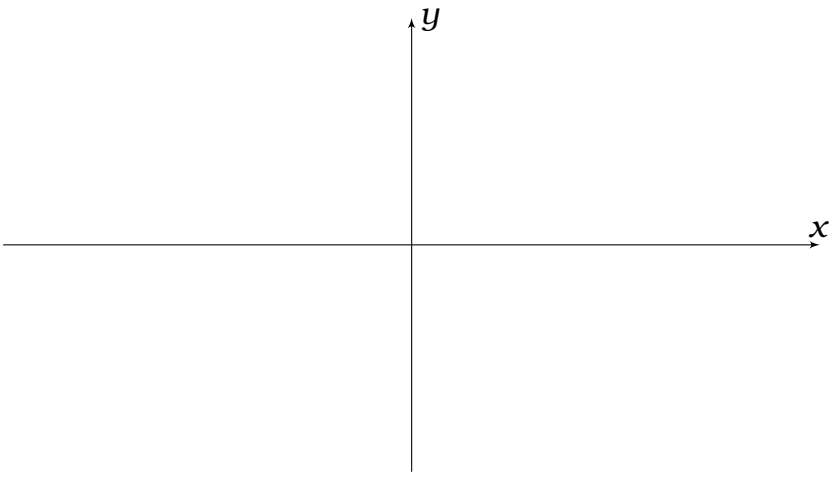
$y = f(x + 2)$

Q20. ***[2 marks]***
Sketch the graph of

$$y = x^2(x - 1)(x - 2)$$

and hence or otherwise solve the inequality

$$x^2(x - 1)(x - 2) < 0.$$



Q21. ***[3 marks]***
Which of the numbers is larger?

Q21.1. $\log_{(a2)}(a3)$ or $\log_{(a3)}(a2)$

Q21.2. $\left(\frac{1}{2}\right)^x$ or $\left(\frac{1}{2}\right)^{x+1}$

Q21.3. $\log_a b$ or $\log_{(2a)} b$ when $a > 1$

Q22. ***[1 marks]***
Find the distance of the centre of the circle $(x + 6)^2 + (y - 8)^2 = 11$ from the origin.

Q23. ***[3 marks]***
Find the coordinates of the points where the circle $(x-2)^2 + (y + 8)^2 = 100$ intersects the x-axis.

Q24. ***[2 marks]***
What is the constant term of the expansion $\left(x + \frac{2}{x}\right)^6$?

DRAFT